1 (1pt.) DESCRIBE IN WORDS WHERE EQL. EQ's COME FROM

Beginning w/ Newton's 2nd law, $\Sigma \underset{\sim}{F}$ is given by the sum of surface forces and body forces. Surface forces are due to integration of normal forces $\underset{\sim}{n} \cdot \underset{\approx}{\sigma}$ OVER THE SURFACE, WHICH IS EQUIVALENT TO divergence of stress integrated over the volume. Further conservation of mass reduces LHS OF EQUATION. (GRADE LENIENTLY).

2 (3pt) CSA $1 m^2$, FORCE 1N in x_1 direction

A) $\sigma_{11} = 1\, N/m^2 = 1\, Pa$

B) $\sigma_{11} = E\,\epsilon_{11} \Rightarrow \epsilon_{11} = \dfrac{\sigma_{11}}{E}$

$$\epsilon_{11} = \frac{1\, Pa}{1 \times 10^{6}\, Pa}$$

C) $\epsilon_{22} = \epsilon_{33} = -\nu\,\epsilon_{11}$

$\nu = 0.5$ FOR INCOMP. LINEAR ELASTIC, ISOTROPIC SOLID

B/C NO SHEAR WE ARE ALREADY IN THE PRINCIPAL AXES AND

$\epsilon_1 = 1 \times 10$

$\epsilon_2 = \epsilon_3 = -5 \times 10^{-7}$

$\boxed{3}$ (2 pts) How many Eq's is Eq. 5.4? Write out.

It's 9: $i = 1, 2,$ or 3. $j = 1, 2,$ or 3.

$$\overset{i=1,2,3 \text{ (not summed)}}{\sigma_{ii}} = \frac{E}{(1+\nu)}\left[\epsilon_{ii} + \frac{\nu}{1-2\nu}\left(\epsilon_{11}+\epsilon_{22}+\epsilon_{33}\right)\right]$$

$$\overset{i \neq j}{\sigma_{ij}} = \frac{E}{(1+\nu)}\epsilon_{ij}$$

$\boxed{4}$ (1 pt) 5 examples of tissues subject to shear

Skin, rotator cuff, cartilage, endothelium, intestinal epithelium, bone (fluid flow) etc.

$\boxed{5}$ (1 pt) Name/explain 5 bad assumptions of our artery model

Arterial wall is nonlinear
has residual stress
has axial stress
experiences shear
is viscoelastic
subject to external loads

etc.

6 (2pts) FROM GENERAL SOLUTION 8.9, PROVE 8.10

$$u(r) = C_1 r + C_2 \frac{1}{r}$$

$$\frac{du}{dr} = C_1 - C_2 \frac{1}{r^2}$$

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left(\frac{du}{dr} + \nu \frac{u}{r}\right)$$

$$= \frac{E}{1-\nu^2}\left(C_1 - \frac{C_2}{r^2} + \nu\left(C_1 + C_2 \frac{1}{r^2}\right)\right)$$

$$\sigma_{rr}(r_o) = 0 = C_1 - \frac{C_2}{r_o^2} + \nu C_1 + \nu \frac{C_2}{r_o^2}$$

$$0 = C_1(1+\nu) + \frac{C_2}{r_o^2}(\nu - 1)$$

$$\boxed{C_1 = \frac{C_2(1-\nu)}{r_o^2(1+\nu)}}$$

$$\sigma_{rr}(r_i) = -p_i = \frac{E}{1-\nu^2}\left(C_1 - \frac{C_2}{r_i^2} + \nu C_1 + \nu \frac{C_2}{r_i^2}\right)$$

$$= \frac{E}{(1+\nu)(1-\nu)}\left[C_1(1+\nu) - \frac{C_2}{r_i^2}(1-\nu)\right]$$

$$= E\left[\frac{C_1}{(1-\nu)} - \frac{C_2}{r_i^2(1+\nu)}\right]$$

$$= E\left[\frac{C_2\cancel{(1-\nu)}}{r_o^2(1+\nu)}\frac{1}{\cancel{(1-\nu)}} - \frac{C_2}{r_i^2(1+\nu)}\right]$$

$$-p_i = \frac{E C_2}{(1+\nu)}\left[\frac{1}{r_o^2} - \frac{1}{r_i^2}\right]$$

$$C_2 = \frac{p_i(1+\nu)}{E\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right]}$$

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left[\frac{du}{dr} + \nu\frac{u}{r}\right]$$

$$= \frac{E}{1-\nu^2}\left[C_1 - \frac{C_2}{r^2} + \nu C_1 + \nu\frac{C_2}{r^2}\right]$$

$$= \frac{E}{1-\nu^2}\left[C_1(1+\nu) - \frac{(1-\nu)C_2}{r^2}\right]$$

$$= \frac{E}{1-\nu^2}\left[\frac{C_2(1-\nu^2)}{r_o^2(1+\nu)} - \frac{(1-\nu)}{r^2}C_2\right]$$

$$= \frac{E C_2}{1-\nu^2}\left[\frac{(1-\nu^2)}{r_o^2(1+\nu)} - \frac{(1-\nu)}{r^2}\right]$$

$$= \frac{\cancel{E}\, p_i(1+\nu)}{\cancel{E}\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)(1-\nu^2)}\left[\frac{1-\nu^2}{r_o^2(1+\nu)} - \frac{(1-\nu)}{r^2}\right]$$

$$= \frac{p_i}{\left(\frac{1}{r_i^2} - \frac{1}{r_i^2}\right)(1\cancel{-\nu^2})}\left[\frac{1\cancel{-\nu^2}}{r_o^2} - \frac{1\cancel{-\nu^2}}{r^2}\right]$$

$$\sigma_{rr} = \frac{p_i}{\frac{1}{r_i^2} - \frac{1}{r_o^2}}\left[\frac{1}{r_o^2} - \frac{1}{r^2}\right]$$

$$= \frac{p_i r_i^2}{1 - \frac{r_i^2}{r_o^2}}\left[\frac{1}{r_o^2} - \frac{1}{r^2}\right]$$

$$\boxed{\sigma_{rr} = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right]}$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}\left(\frac{u}{r} + \nu\frac{\partial u}{\partial r}\right)$$

$$= \frac{E}{1-\nu^2}\left[C_1 + \frac{C_2}{r^2} + \nu C_1 - \frac{\nu C_2}{r^2}\right]$$

$$= \frac{E}{1-\nu^2}\left[C_1(1+\nu) + \frac{C_2}{r^2}(1-\nu)\right]$$

$$= \frac{E}{1-\nu^2}\left[\frac{C_2(1-\nu)(1+\nu)}{r_o^2(1+\nu)} + \frac{C_2}{r^2}(1-\nu)\right]$$

$$= \frac{E\,C_2}{1-\nu^2}\left[\frac{1-\nu^2}{r_o^2(1+\nu)} + \frac{1-\nu}{r^2}\right]$$

$$= \frac{\cancel{E}\,p_i(1+\nu)}{\cancel{E}(1-\nu^2)\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right]}\left[\frac{1-\nu^2}{r_o^2(1+\nu)} + \frac{1-\nu}{r^2}\right]$$

$$= \frac{p_i}{\cancel{(1-\nu^2)}\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right)}\left[\frac{\cancel{1-\nu^2}}{r_o^2} + \frac{\cancel{1-\nu^2}}{r^2}\right]$$

$$= \frac{p_i}{\frac{1}{r_i^2} - \frac{1}{r_o^2}}\left(\frac{1}{r_o^2} + \frac{1}{r^2}\right)$$

$$= \frac{p_i r_i^2}{1 - \frac{r_i^2}{r_o^2}}\left(\frac{1}{r_o^2} + \frac{1}{r^2}\right)$$

$$\boxed{\sigma_{\theta\theta} = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)}$$

$\boxed{7}$ STRAIN DISPLACEMENT $\to \underset{\approx}{\epsilon}(\underset{\sim}{u})$

INTO CONSTITUTIVE LAW $\sigma(\underset{\approx}{\epsilon}) \to \underset{\approx}{\sigma}(\underset{\sim}{u})$

INTO EQ OF MOTION $(\underset{\sim}{\nabla} \cdot \underset{\approx}{\sigma} + \underline{f} = \rho \underset{\sim}{a})$

PRODUCES A MIXED P.D.E INVOLVING

SPATIAL + TIME DERIVATIVES OF DISPLACEMENT.

SO IN MECHANICS YOU ARE OFTEN GIVEN

FORCES (BOUNDARY CONDITIONS) AND TASKED

WITH PREDICTING DISPLACEMENT.

INDEP VARIABLES $\underset{\sim}{x}$ and maybe t

DEPENDENT " $\underset{\sim}{u}$